Azonosító jel: | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA ● 2015. május 5.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2015. május 5. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 240 minutes. When that time is over, you will have to stop working.

2. You may solve the problems in any order.

3. In Section II, you are only required to solve four out of the five problems**. When you have finished the examination, write in the square below the number of the problem NOT selected.**
   If it is *not clear* for the examiner which problem you do not wish to be assessed, then problem 9 will not be assessed.

4. In solving the problems, you are allowed to use a calculator that cannot store and display verbal information. You are also allowed to use any book of four-digit data tables. The use of any other electronic device, or printed or written material is forbidden.

5. **Always write down the reasoning used in obtaining the answers, since a large part of the attainable points will be awarded for that.**

6. **Make sure that the calculations of intermediate results are also possible to follow.**

7. In solving the problems, theorems studied and given a name in class (e.g. the Pythagorean theorem or the altitude theorem) do not need to be stated precisely. It is enough to refer to them by the name, but their applicability needs to be briefly explained. Reference to other theorem(s) will only be awarded full mark if the theorem and all its conditions are stated correctly (proof is not required), and the applicability of the theorem to the given problem is explained.

8. Always state the final result (the answer to the question of the problem) in words, too.

9. Write in pen. Diagrams are also allowed to be drawn in pencil. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

10. Only one solution to each problem will be assessed. In the case of more than one attempt to solve a problem**, indicate clearly** which attempt you wish to be marked.

11. Do not write anything in the grey rectangles.

I

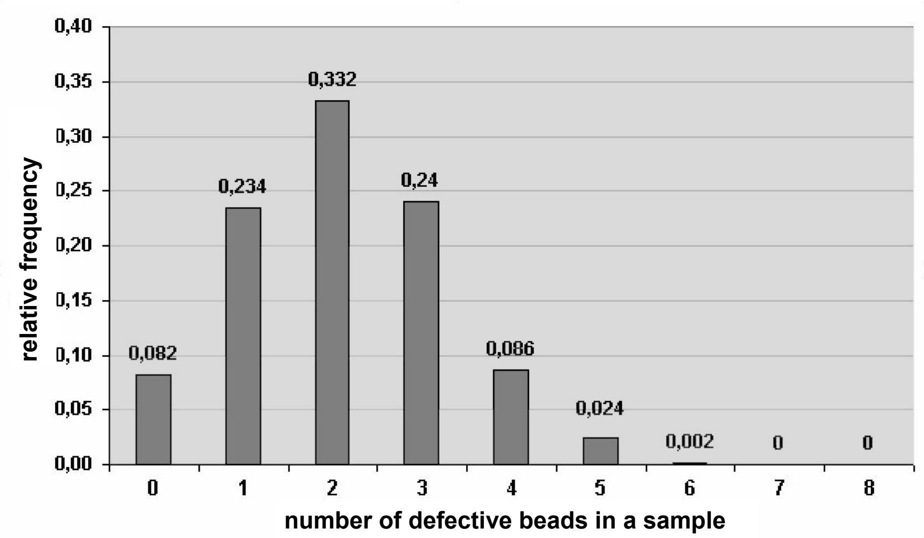
1. Given the circle k of equation $4x^2 + 4y^2 = 90$ and the line g of equation $x + 3y = 0$, determine the equations of the tangents to circle k that are parallel to line g.

T.:	12 points	

2. A box contains 40 glass beads, 8 of which are defective. An experiment consists in selecting a random sample of 10 beads **without replacement**, and counting the number of defective beads among them.

a) A group of students carried out the above experiment 500 times. After the experiments, they did some statistics with their findings: they represented in a bar chart the relative frequencies of faulty beads in the 10-element samples. Use the diagram to answer the following questions:

I. What was the largest number of defective beads occurring in a sample?
II. What was the most frequently occurring number of defective beads in a sample?
III. How many times was there no defective bead at all in the sample?



b) Calculate the probability that if the experiment is carried out once, the sample will contain exactly 2 defective beads. Consider the relative frequency of this event obtained from the 500 experiments represented above. What percentage of the calculated probability is it?

c) In another experiment, 10 beads are selected from the same 40 beads **by sampling with replacement**. What is the probability that the sample will contain exactly 2 defective beads in this case?

a)	4 points	
b)	5 points	
c)	4 points	
T.:	13 points	

3. We want to determine the heights of a hill and of a lookout tower built on the top of the hill. Let A denote the top of the tower, and let B denote the base of the tower. There is a horizontal flat plain at the foot of the hill. We mark the points P and Q lying at a distance of 30 metres from each other on the plain, in the same direction from the tower. The points A, B, P, and Q are in the same (vertical) plane. The base of the tower (point B) is seen at a 29° angle of elevation from point P, and the top of the tower (point A) is seen at a 33° angle of elevation from P. From point Q, the angle of elevation of the base of the tower is 27°.

a) What is the height, in metres, of the hill above the plain?

b) How tall is the lookout tower?

Round your answers to the nearest metre.

a)	8 points	
b)	5 points	
T.:	13 points	

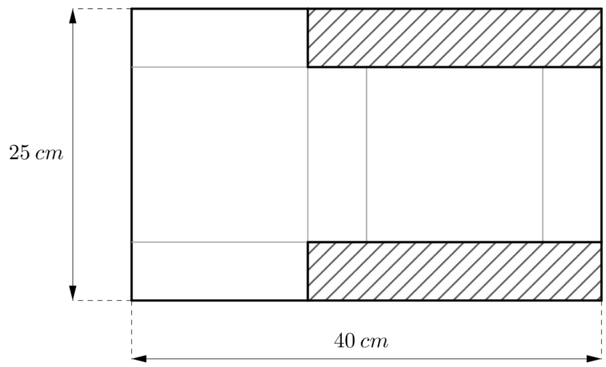
4. Let A denote the set of real solutions of the inequality $4x^2 - 19x + 22 < 0$, and let B denote the set of real solutions of the inequality $\sin 2x < 0$.
Prove that $A \subset B$.

T.:	13 points	

II

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

5. Two congruent rectangles (shaded in the diagram) are cut out of a 40 cm × 25 cm sheet of cardboard. The remaining cardboard is then folded along the edges drawn in the diagram to form a cuboid, such that its height equals the shorter side of the rectangles cut out.



a) What will be the surface area of the resulting cuboid if the shorter side of the rectangles cut out is 2 cm?

b) What should be the length of the shorter side of the rectangles cut out, to maximize the volume of the cuboid formed? What is the maximum volume?

a)	4 points	
b)	12 points	
T.:	16 points	

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6. **a)** The degrees of 8 points in a tree graph on 9 points are known: 1; 1; 1; 1; 2; 3; 3, 3.
Determine the degree of the ninth point.

b) Is there a simple graph on 9 points in which the degrees of the 9 points are all different?

c) In a company of 9 members, people are greeting each other with handshakes. 4 handshakes have taken place so far. In how many different ways may this have happened if no one has shaken hands more than once, and the order of the handshakes does not count?

a)	5 points	
b)	5 points	
c)	6 points	
T.:	16 points	

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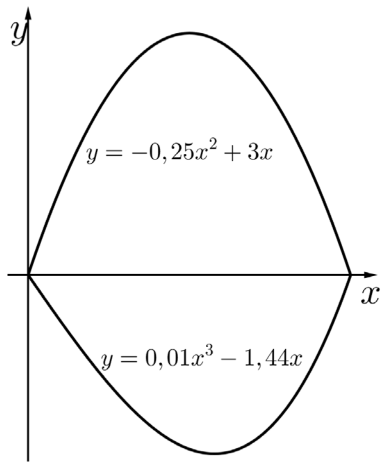
7. In the finals of the chess singles championship organized by a school, every player played every other player exactly once. At the end of the games, it turned out that the scores of the players formed a strictly increasing arithmetic progression.
How many players took part in the finals, and what was the score of the winner if the player finishing in the last place received 1 point altogether? (In chess, 1 point is awarded for winning, 0.5 point for a draw, and 0 points for losing.)

T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

8. Consider those arcs of the curves with equations $y = -0.25x^2 + 3x$ and $y = 0.01x^3 - 1.44x$ for which $0 \le x \le 12$. (The two arcs are shown in the diagram.) It is also given that the common points of the two arcs are the points (0; 0) and (12; 0).



a) For each arc, determine the first coordinate of the point of the arc lying at maximum distance from the *x*-axis.

b) Calculate the area bounded by the two arcs.

c) Define the functions *f* and *g* on the interval]0; 12[by the following rules of assignment:

$f(x) = \dfrac{-0.25x^2 + 3x}{0.01x^3 - 1.44x}$ and $g(x) = -\dfrac{25}{x+12}$.

Prove that $f(x) = g(x)$, and show that the function *g* is strictly increasing.

a)	5 points	
b)	5 points	
c)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 3.

9. The students of a class were given a mathematics test in which they had to solve three problems. The first problem was solved by 22 students altogether, and 16 of these also solved the second problem. Three times as many students solved all three problems as the number of those solving the first problem only. The number of those who solved only the first two problems was two and a half times the number of those solving only the first and third problems.

a) How many students solved all three problems?

30 students took the test altogether. The teacher graded the papers with 1 to 5 integers. The mean of the grades was 3.4, the median was 3.5, and the single mode was 4. When the papers were given back to the students, six of the students taking the test were absent. The grades obtained by the 24 students present were as follows: 7 fives, 5 fours, 6 threes, 4 twos and 2 ones.

b) What may have been the grades of the six students who were absent?

a)	7 points	
b)	9 points	
T.:	16 points	

	number of problem	maximum score	points awarded	maximum score	points awarded
Section I	1	12		**51**	
	2	13			
	3	13			
	4	13			
Section II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	elért pontszám **egész számra** kerekítve/ score rounded to **integer**	programba beírt **egész** pontszám/ **integer** score entered in program
I. rész / Section I		
II. rész / Section II		

javító tanár / examiner

jegyző / registrar

datum / date

datum / date